Loss Aversion, Moral Hazard, and Stochastic Contracts

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Draft: August 1, 2022

I examine whether stochastic contracts benefit the principal under moral hazard and loss aversion. Incorporating the agent's expectation-based loss aversion and allowing for stochastic contracts, I find that stochastic contracts reduce the principal's cost as compared to deterministic contracts. The optimal stochastic contract pays a high wage not only when good signals are realized, but also with a positive probability after the realization of bad signals. The findings have an important implication for designing contracts for loss-averse agents: the principal should insure the agent against wage uncertainty by employing stochastic contracts that increase the probability of a high wage.

JEL: D82, D86, M12, M52

Keywords: loss aversion, moral hazard, stochastic contracts, reference-dependent preferences

* LMU Munich, Department of Economics (Email: hoa.ho@econ.lmu.de). I am tremendously grateful to Klaus Schmidt and Florian Englmaier for their guidance and support. In preparing this draft I have greatly benefited from comments made by David Gill, Benjamin Haeusinger, Paul Heidhues, Fabian Herweg, Heiko Karle, Botond Kőszegi, Matthias Lang, Yves Le Yaouanq, Johannes Maier, Takeshi Murooka, Simeon Schudy, Peter Schwardmann, Lisa Spantig, Roberto Weber, and many seminar participants at the University of Munich, as well as participants at the *Workshop on Belief-Dependent Preference* in Copenhagen (2019), *BRIQ Summer School on Behavioural Economics* in Bonn (2019), *Bavarian Micro Day* in Bayreuth (2019), *RGS Doctoral Conference in Economics* in Dortmund (2020), the *Spring Meeting of Young Economists* (2021), the *Asian Meeting of Econometric Society* (2021), the *Society for Institutional & Organizational Economics* (2021), the *China Meeting of Econometric Society* (2021), the *European Association for Research in Industrial Economics* (2021), the *Annual Conference of the German Economic Association* (2021), and the *ASSA/AEA Annual Meeting* (2022). I gratefully acknowledge the funding provided through the International Doctoral Program "Evidence-Based Economics" of the Elite Network of Bavaria and the Deutsche Forschungsgemeinschaft through CRC TRR 190.

The interplay between risk aversion and incentives is central to the moral hazard literature. In this literature, one of the very few general results, as pointed out by Bolton and Dewatripont (2005) , is the informativeness principle. This principle, going back to Holmstrom et al. (1979), Holmstrom (1982), and Grossman and Hart (1983), states that a wage contract should contain only informative signals about the agent's effort. However, many labor contracts are stochastic in that they include noise that does not provide any statistical information about the agent's effort.[1] This gap between theory and observed contracts suggests that a traditional approach focusing solely on risk aversion might give a partial and incomplete picture of the moral hazard problem.

To fill this gap, this article incorporates loss aversion in the moral hazard problem. Although loss aversion is a fundamental concept in behavioral economics and is well-established with ample experimental and field evidence, the interplay between loss aversion and incentives remains understudied in the moral hazard literature. More recently, Camerer, Loewenstein and Rabin (2004) argue that loss aversion drives much of behavior. "In a wide variety of domains", as Rabin (2004) states, "people are significantly more averse to losses than they are attracted to same-sized gains". One prominent realm in which loss aversion plays a significant role is the domain of money and wealth (Tversky and Kahneman, 1991). It is thus important to incorporate loss aversion in the analysis of the optimal wage contract and to better understand how loss aversion affects the trade-off between insurance and incentives in the moral hazard model.

This article analyzes the optimal wage contract in a setting of moral hazard and loss aversion, where the agent is expectation-based loss averse and the principal can use stochastic contracts. The main result is that stochastic contracts reduce the principal's cost to implement a given action in comparison to deterministic contracts. When performance signals are highly informative about the agent's effort, the dominance of stochastic contracts over deterministic contracts holds for almost any degree of loss aversion. However, the second-best stochastic contract might not exist as the principal prefers to insure the loss-averse agent to the

[1] In workplaces, firms successfully adopt teams and team incentives (Che and Yoo, 2001; Lazear and Shaw, 2007; Bandiera, Barankay and Rasul, 2013) in which a team's performance depends not only on an employee's effort but also on the effort exerted by other team members. In addition, non-executive employees increasingly receive payments in stock options (Core and Guay, 2001; Bergman and Jenter, 2007; Hochberg and Lindsey, 2010; Kim and Ouimet, 2014) whose valuation is influenced by external shocks in the financial sector.

greatest extent possible. To mitigate this non-existence issue, I find that limited liability ensures the existence of the optimal contract. The optimal stochastic contract is characterized as follows: it pays a high wage with certainty when a good signal is realized and with a positive probability when a bad signal is realized.

More specifically, I extend the simple principal-agent model under moral hazard, in which both the agent's actions and observable signals are binary, by adding two features. First, the agent is expectation-based loss averse as defined in Kőszegi and Rabin (2006, 2007). In particular, the agent forms a reference point *after* taking an action, and thus his chosen action affects his reference point. The agent compares his realized wage to the stochastic reference point, and he feels a loss if the actual wage is smaller than the reference wage. Second, the principal can add noise to performance signals by employing stochastic contracts. In particular, the principal can add a lottery after observing the realized signal. Stochastic contracts thus serve as a tool for the principal to manipulate the signal distribution. A crucial feature of my model is that the principal can fully control the structure of the stochastic contract, that is, the odds of the lottery.

I find that there exists a stochastic contract that strictly dominates any deterministic contracts under a weak condition. In the stochastic contract, the principal pays out a high wage whenever she observes a good signal, while upon observing a bad signal she adds a lottery that gives either the same high wage or a low wage that serves as a harsh penalty on the agent for the bad signal. This stochastic feature is beneficial for two reasons. First, the stochastic contract with this turning-a-blind-eye structure remedies an implementation problem associated with loss aversion. In deterministic contracts, this implementation problem is well-established; that is, the agent may choose the stochastically dominated action when he is sufficiently loss averse (Herweg, Müller and Weinschenk, 2010). As a result, the principal may be unable to induce the agent to exert effort. In sharp contrast, by employing the stochastic contract, the principal can always implement the desired action for any degree of loss aversion.

Second, even if deterministic contracts do not face the implementation problem, the stochastic contract helps the principal lower the cost of implementing the desired action beyond what is achieved under the optimal deterministic contract. Note that the stochastic contract, as compared to deterministic contracts, has two opposing effects on the principal's cost. On the one hand, the stochastic contract might increase the principal's cost, because the high wage is now paid out more often and a larger wage spread is required to incentivize the agent to work. On the other hand, the stochastic contract reduces the probability of the agent feeling a loss, and thus the principal might capitalize on this reduction in the agent's loss premium to achieve a lower cost. When the positive effect of reducing the loss premium outweighs the negative effect of increasing the expected bonus, the stochastic contract dominates deterministic contracts. Whether the stochastic contract is dominant depends on the agent's degree of loss aversion and the informativeness of performance signals.

Interestingly, as performance signals get more informative about the agent's action, the principal favors the stochastic contract under a wider range of degrees of loss aversion. When performance signals are highly uninformative, the principal is better off with the stochastic contract under the most restrictive condition, that is, only when the agent feels losses at least twice as strongly as same-sized gains. This condition gets weaker if performance signals provide some information about the agent's action. When performance signals convey almost perfect information, the stochastic contract dominates deterministic contracts for almost any degree of loss aversion. Intuitively, when performance signals are highly informative, the principal can provide further wage certainty at a negligible cost. Thus, this finding has an important implication for designing contracts for loss-averse agents: the principal has an incentive to add noise after the bad signal to insure the agent against wage uncertainty.

Yet I show that the second-best optimal stochastic contract might not exist. In particular, the principal's cost strictly decreases as the probability of getting the high wage increases. This implies that the principal prefers to push the probability of the high wage close to one and punish the agent very harshly when the worst outcome is realized. However, the principal cannot provide wage certainty because of the incentive constraint, and hence the solution to the principal's problem is not well-defined. This existence problem differs from the above implementation problem under loss aversion in that a stochastic contract can always implement the desired action but an optimal stochastic contract might not exist. Given the wide range of degrees of loss aversion under which stochastic contracts dominate deterministic contracts, the existence problem appears more severe than previously thought.

In mitigating this issue, I find that the non-existence problem disappears if the agent is protected by limited liability. The optimal stochastic contract pays a bonus with certainty when the good signal is realized and with a positive probability when the bad signal is realized; otherwise, the agent receives the lowest possible wage, at which the limited liability constraint is binding. This finding highlights the importance of imposing limited liability in stochastic contracts to restrict the extent to which the principal can punish the agent in the event of a bad signal and to ensure that the second-best optimal contract exists.

Lastly, I consider the general case when the agent is both risk and loss averse and show that for any given degree of relative risk aversion, there exists a sufficiently large degree of loss aversion such that the principal still prefers to add noise in the contract. Intuitively, the principal faces a trade-off when implementing a stochastic contract: risk aversion imposes an additional cost of adding noise, but loss aversion implies that the principal still benefits from a reduced probability of loss. The principal strictly prefers a stochastic contract over any deterministic contracts when the benefit of noise outweighs its cost, that is, when loss aversion plays a more important role than risk aversion in the agent's preference.

So far, I have assumed that a reference point is formed after the decision is taken and I allow for a stochastic reference point. In Section V, I discuss alternative notions of loss aversion. In particular, the result holds under forward-looking disappointment aversion according to Bell (1985), Loomes and Sugden (1986), or Gul (1991), in which the reference point is the recent expectation but does not allow for stochastic reference points. It also remains valid under the concept of *preferred personal equilibrium* by Kőszegi and Rabin (2007), which assumes that the reference point is formed before taking the decision and hence is taken as given. The robustness of the result suggests that when loss aversion plays a significant role in the agent's preference, noise should be generally added to performance signals in the optimal contract. Perhaps paradoxically, by employing stochastic contracts and manipulating the noise structure, the principal can insure the loss-averse agent against wage uncertainty.

How the principal can commit to stochastic contracts is an important concern in practice. There are several ways in which the principal can credibly engage in stochastic contracts that often ignore the agent's bad performance. First, a fixed wage contract with firing threats—which remains the most commonly used contract in practice—is one example of such stochastic contracts. The firing threats, which happen only in rare instances and do not occur every time the agent performs poorly, enable the principal to overlook the agent's bad performance. Second, the principal can tie payments to stock options and exploit the randomness of stock options. Jenter and Kanaan (2015) examine CEO turnover and find that CEOs are fired after bad firm performance (i.e., bad stock price) only when the market also performs badly—a factor that is beyond their control—but not when the market performs well. This empirical evidence is consistent with my theoretical prediction. Finally, team incentives, as suggested by Daido and Murooka (2016), can also be thought of as a stochastic contract: if some team members succeed, a high wage is paid out to both high and low performing team members, whereas only in the worst case scenario when all team members fail is a low wage paid out.

The rest of the article is organized as follows. Section I summarizes the related literature. Section II outlines the model, and Section III specifies the principal's problem and derives the set of feasible contracts. Section IV presents the main results and characterizes the optimal contract. Section V discusses a general case of loss aversion and risk aversion, and considers alternative notions of loss aversion. Section VI concludes. All proofs of lemmas and propositions are relegated to the Appendix.

I. Related Literature

This article relates to the extensive literature on reference-dependent preferences, starting out with the seminal work of Kahneman and Tversky (1979) where the agent's utility depends on a reference point and the agents feel losses more strongly than gains. Subsequently, as reviewed by Barberis (2013), several articles have contributed to theoretical extensions—covering reference-dependent models of both static (Bell, 1985; Loomes and Sugden, 1986; Munro and Sugden, 2003; Sugden, 2003; Kőszegi and Rabin, 2006; De Giorgi and Post, 2011) and dynamic nature (Barberis and Huang, 2001; Barberis, Huang and Santos, 2001; Kőszegi and Rabin, 2009)—and applications of reference-dependent preferences to real-life problems, such as in tournaments (Gill and Stone, 2010), saving decisions (Jofre, Moroni and Repetto, 2015), renegotiation (Herweg and Schmidt, 2015; Herweg, Karle and Müller, 2018), cheating behavior (Grolleau, Kocher and Sutan, 2016), asset pricing (Pagel, 2016), life-cycle consumption (Pagel, 2017), contract preferences (Imas, Sadoff and Samek, 2017), intertemporal incentives (Macera, 2018), portfolio choices (Pagel, 2018), school choice (Dreyfuss, Heffetz and Rabin, 2019; Meisner and von Wangenheim, 2019), student performance (Karle, Engelmann and Peitz, 2020), voting abstention (Daido and Tajika, 2020), and quality disclosure (Zhang and Li, 2021). My article contributes to the literature strand that incorporates expectation-based reference-dependent preferences into moral hazard models, as summarized by Koszegi (2014), by providing the characteristics of the optimal stochastic contract for loss-averse agents.

My results speak to a growing literature that highlights the optimality of noise in the contract. The dominance of stochastic contracts has predominantly been associated with complex contracting environments (e.g., screening model, dynamic interaction) and the presence of additional constraints (e.g., aspiration constraints, subjective performance signals). Haller (1985) finds that randomization benefits the principal when the agent faces an aspiration constraint of achieving certain income levels with certain probabilities. Strausz (2006) shows that stochastic mechanisms may be optimal in a screening context. Lang (2020) examines the optimal contract with subjective evaluations and shows that stochastic contracts may increase the principal's profits and eliminate the requirement for a third-party payment. Ostrizek (2020) studies a dynamic principal-agent setting and finds that the principal prefers to set wages contingent on a noisy information structure, because the agent remains uninformed about their match-specific ability and can be motivated more cheaply. Contributing to this literature, I show that noise can be beneficial even in a simple moral hazard model with loss aversion: the principal can reduce the implementation cost by manipulating noise to insure the loss-averse agent against wage uncertainty.

My article is most closely related and complementary to Herweg, Müller and Weinschenk (2010) who are the first to show that, in the setting of moral hazard and loss aversion, the principal prefers to lump together different signals and the optimal deterministic contract is a bonus contract. Extending their findings, my article provides further insight into the characteristics of the optimal contract under loss aversion: the probability of getting a bonus is set as high as possible. There are three important differences between Herweg, Müller and Weinschenk (2010) and this article. First, for the greater part of the analysis, they restrict their attention to deterministic contracts, whereas I allow the principal to employ stochastic contracts. Second, they propose stochastic contracts as a remedy for the implementation problem associated with deterministic contracts. I show that stochastic contracts can achieve more than a remedy: stochastic contracts help reduce the principal's cost, even when deterministic contracts can implement the desired action. Third, their article suggests the dominance of stochastic contracts if the degree of loss aversion is sufficiently large, whereas my article highlights a condition under which stochastic contracts can be optimal for almost any degree of loss aversion.

In the literature on behavioral contract theory, this article also relates to Daido and Murooka (2016) who show that the principal may prefer team incentives when the agents are loss averse. Similar to their article, my article emphasizes the benefit of adding noise to individual performance signals. Whereas their article focuses on team incentives and takes the team structure as given, I examine individual stochastic contracts and consider noise as one of the principal's control variables. In the earlier version of their article, Daido and Murooka (2013) consider the moral hazard and loss aversion problem under limited liability. On the one hand, my article confirms their findings—limited liability ensures the existence of the second-best contract—and characterizes the optimal stochastic contract. On the other hand, I extend their findings by focusing on a general setting without limited liability and providing further insight: without the limited liability constraint, the principal prefers to stochastically compensate for the agent's low performance to the largest possible extent, even when the degree of loss aversion is small.

II. The Model

I consider a principal-agent model in a moral hazard and loss aversion setting. The principal (she) offers a one-period employment contract to the agent (he), which the agent either accepts or rejects. If the agent rejects it, he receives his reservation utility which is assumed to be zero.[2] If the agent accepts the contract, he then takes a binary action $a \in \{a_H, a_L\}$; that is, he either "works" $(a = a_H)$ or "shirks" $(a = a_L)$. The cost of working for the agent is $c(a_H) = c$, for $c > 0$,

[2]The assumption that the reservation utility is zero is consistent with the "quitting" constraint. This assumption is made for the sake of simplicity of analysis. The main results would continue to hold when the reservation utility is positive.

and the cost of shirking is normalized at zero $c(a_L) = 0$.

The action a is private information of the agent that the principal cannot observe. Instead, the principal is assumed to observe a contractible signal of the agent's action. The signal $s \in S = \{1, 2\}$ is good ($s = 2$) or bad ($s = 1$).[3] The agent receives the good signal with probability q_H if he works and with probability q_L if he shirks, where $1 > q_H > q_L > 0$. The signal distribution is common knowledge.

The agent exhibits expectation-based loss aversion as defined in Kőszegi and Rabin (2006, 2007). The agent's utility has two additively separable components: the standard "consumption utility" and the reference-dependent "gain–loss utility". The agent's consumption utility, denoted by $u(\cdot)$, is assumed to be strictly increasing, (weakly) concave, and unbounded, that is, $u'(\cdot) > 0$ and $u''(\cdot) \leq 0$. The second component comes from reference-dependent preferences: the agent compares a realized outcome to a stochastic reference point, and how his overall utility is affected depends on whether this comparison is perceived as a gain or a loss. The gain–loss function $\mu(\cdot)$ satisfies the assumptions on the "value function" by Tversky and Kahneman (1991). I assume that the gain–loss function is piecewise linear,

$$\mu(m) = \begin{cases} m & \text{if } m \geq 0 \\ \lambda m & \text{if } m < 0 \end{cases}$$

where $\lambda \geq 1$ represents the degree of loss aversion and m is the difference between a realized outcome and an expected outcome.

To determine the reference point, I apply the concept of choice-acclimating personal equilibrium (CPE) in the sense of Kőszegi and Rabin (2007), which makes two important assumptions. First, the agent forms the reference point, to which realized outcomes are evaluated, *after* making the decision, and thus his decision affects his reference point. As mentioned by Kőszegi and Rabin (2007), CPE considers outcomes that are resolved long after all decisions are made. Thus, the reference point is endogenously determined as the agent's rational expectation about the outcomes given his decision. Second, the reference point is stochastic if the decision's outcome is stochastic. To form a stochastic reference point, it is

[3] I will discuss the case of finite signals in Section V and show that the main results are robust under the assumption of finite signals.

assumed that the agent knows the set of possible outcomes and its probability distribution conditional on his decisions. These two assumptions give rise to a crucial feature of CPE: a stochastic outcome is evaluated to a stochastic reference point by comparing outcome by outcome, where each comparison is weighted with the joint probability with which a certain outcome is realized and an alternative outcome is expected.

On the other hand, the principal is assumed to be risk and loss neutral. I assume that the agent's "work" generates sufficient profit for the principal that she strictly prefers to implement the high action a_H. Thus, I focus on the principal's cost minimization problem and examine the optimal contract design under moral hazard with loss aversion.

In designing the optimal contract, the principal can distort the outcome distribution by adding noise to the performance signals. Put differently, she can fully employ stochastic contracts to implement the desired action. A stochastic contract specifies wage payments contingent not only on the contractible signals but also on a stochastic device that does not depend on the agent's action. Formally, the principal offers the agent a state-contingent stochastic contract $(\mathcal{C}_s)_{s\in S}$, in which each $\mathcal{C}_s$ entails a stochastic device—uncorrelated with the agent's action—that specifies wage payments within the contract.

In the setting of two signals, the principal offers a stochastic contract $(\mathcal{C}_1, \mathcal{C}_2)$. If the principal observes the good signal $s = 2$, then the agent receives $\mathcal{C}_2$, which specifies a lottery $(p_2, 1-p_2)$ over wage payments.[4] Analogously, $\mathcal{C}_1$ with a lottery $(p_1, 1-p_1)$ is realized if the bad signal $s = 1$ is observed. Importantly, the principal has full control over the design of these lotteries (p_1, p_2), which are referred to as the "stochastic structure".

As shown in Figure 1, the distribution of the outcomes $i \in \{1, \dots, 4\}$ depends on both the agent's action and the principal's choice of stochastic structure. Figure 1 represents how the distribution of the wage payments $(w_i)_{i=1}^4$ depends on the agent's action $a \in \{a_H, a_L\}$ under the stochastic contract. By committing to the stochastic structure (p_1, p_2) in the contract, the principal makes the wage distribution common knowledge to the agent before he chooses his action. Thus, in the process of choosing an action, the agent incorporates the structure of the

[4] The assumption that a lottery specifies two outcomes is without loss of generality. Even when the lottery specifies more than two outcomes, the principal prefers to lump outcomes into two distinct sets. This is in line with the finding by Herweg, Müller and Weinschenk (2010) that the optimal contract specifies two levels of wages.

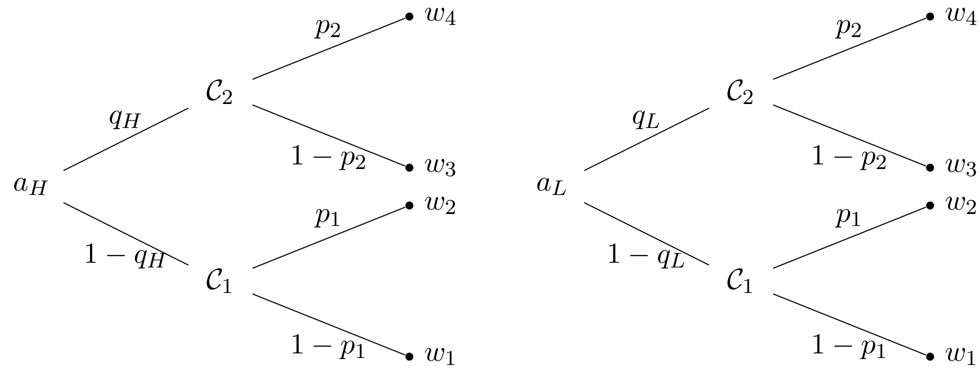

Figure 1. : Distribution of wage payments under stochastic contracts

Note: The left (right) diagram depicts the distribution of wage payments conditional on the agent's high (low) action a_H (a_L).

stochastic contract and forms a rational expectation about monetary outcomes.

More precisely, consider a particular case in which the agent chooses the high action a_H and a certain outcome i is realized. The agent receives w_i and incurs effort cost c. Given that w_i is realized, he compares the realized outcome w_i to all alternative outcomes. Although w_i is realized, he expects an alternative outcome $j \neq i$ to be observed with some probability $f_j(a_H)$. If $w_i > w_j$, the agent experiences a gain of $u(w_i) - u(w_j)$, whereas if $w_i < w_j$, the agent experiences a loss of $\lambda(u(w_i) - u(w_j))$. If $w_i = w_j$, there is no gain or loss involved. The agent's utility in this particular case is given by

$$u(w_i) + \sum_{j|w_i > w_j} f_j(a_H)(u(w_i) - u(w_j)) + \sum_{j|w_i < w_j} f_j(a_H)\lambda(u(w_i) - u(w_j)) - c$$

Notice that this particular comparison occurs with the probability $f_i(a_H)$ that outcome i is realized. When there is uncertainty in the decision's outcome, the agent's expected utility is obtained by averaging over all possible comparisons.

III. The Principal's Problem

Denote $u_i = u(w_i)$. With this notation, the agent's expected utility from choosing action $a \in \{a_H, a_L\}$ is given by

$$EU(a) = \sum_i f_i(a)u_i - (\lambda - 1)\sum_i \sum_{j|u_i > u_j} f_i(a)f_j(a)(u_i - u_j) - c(a)$$

The first term captures the agent's expected consumption utility. For $\lambda = 1$, we have the standard case without loss aversion. The second term captures the gain–loss utility. While the agent expects a high wage u_i to occur with probability $f_i(a)$, he receives a low wage u_j with probability $f_j(a)$ and experiences a loss of $\lambda(u_i - u_j)$. On the other hand, if the agent expects the low wage with probability $f_j(a)$ and receives the high wage with probability $f_i(a)$, he experiences a gain of $u_i - u_j$. Because losses loom larger than gains of equal size ($\lambda \geq 1$), the gain–loss utility is always negative in expectation. Following Herweg, Müller and Weinschenk (2010), I refer to this expected net loss as the agent's "loss premium". For an agent with a higher degree of loss aversion, the principal has to pay a higher loss premium in a given contract.

Let $h(\cdot) := u^{-1}(\cdot)$ be the wage that the principal offers the agent to obtain utility u_i, that is, $h(u_i) = w_i$. Due to the assumptions on $u(\cdot)$, $h(\cdot)$ is strictly increasing and (weakly) convex. Following Grossman and Hart (1983), I regard $\mathbf{u} = (u_1, \ldots, u_4)$ as the principal's control variables in her cost minimization problem. The principal specifies a wage payment w_i for each outcome i in the employment contract, equivalently utility u_i.

The key assumption is that, besides the wage payments, the principal controls the stochastic structure $\mathbf{p} = (p_1, p_2)$. In sharp contrast to deterministic contracts, stochastic contracts allows the principal to manipulate the outcome distribution. Her problem is thus to minimize the expected wage payment that implements a_H subject to the participation and incentive compatibility constraints.

$$\min_{\mathbf{u},\mathbf{p}} \; E(h(u_i))$$

$$\text{(PC)} \qquad \text{subject to} \quad EU(a_H) \geq 0$$

$$\text{(IC)} \qquad EU(a_H) \geq EU(a_L)$$

In deterministic contracts, it is well-established that if the agent is sufficiently loss averse, that is, $\lambda > 2$, then the agent might choose the stochastically dominated action, and the principal, facing a severe implementation problem, might be unable to induce the high action (Herweg, Müller and Weinschenk, 2010). I now examine whether there are incentive-compatible wage payments under stochastic contracts to implement a_H and show that, in sharp contrast to deterministic contracts, stochastic contracts do not suffer from this implementation problem.[5]

LEMMA 1: *Suppose $u''(\cdot) \leq 0$ and $\lambda \geq 1$. For every λ, there exists a stochastic contract such that the action a_H can be implemented.*

Lemma 1 states that the principal can always implement the desired action with a stochastic contract. For *any* given degree of loss aversion, there exist incentive-compatible wages and a stochastic structure such that the agent accepts the stochastic contract and chooses the high action. In particular, the principal pays out a high wage whenever she observes a good signal, while after observing a bad signal she adds a lottery that gives either the high wage or a low wage. This means, in the stochastic contract, that the principal turns a blind eye when the agent receives a bad signal and insures the agent against wage uncertainty. The stochastic contract circumvents the implementation problem of deterministic contracts, because, by increasing the probability of getting the high wage, the principal reduces the agent's expected net loss much more when the agent works than when he shirks. For a loss-averse agent, whose concern is to minimize the expected net loss, the stochastic contract makes working more attractive than shirking.

So far, it is established that the constraint set of the principal's cost minimization problem is non-empty for the high action a_H given any degree of loss aversion. I restrict attention to the stochastic contract of the turning-a-blind-eye structure for the following analysis.[6]

IV. The Optimal Contract

In this section, I examine the existence and the characteristics of the optimal contract. First, I focus on the case of a loss-averse but risk-neutral agent. I will show that under a weak condition there exists a stochastic contract that strictly dominates any deterministic contracts. The principal can lower the cost of implementing the desired action by employing stochastic contracts rather than deterministic contracts. Surprisingly, this holds true even when deterministic

[5] All proofs of lemmas and propositions are provided in the Appendix.

[6] The strategy of "turning a blind eye" was first discussed in Herweg, Müller and Weinschenk (2010), who show that when facing an implementation problem, the principal can indeed still implement the desired action by stochastically ignoring the agent's bad performance. In this article, I focus on the situations in which the implementation problem does not arise and the principal can use deterministic contracts to induce the agent to work.

contracts do not face the implementation problem. The dominance of stochastic contracts, however, implies that for many cases the second-best optimal stochastic contract does not exist. With agents being expectation-based loss averse, an existence problem, which does not exist in the standard model, arises. Second, I examine whether limited liability mitigates the non-existence issue of stochastic contracts and characterize the second-best optimal stochastic contract.

A. Strict Dominance of Stochastic Contracts

Consider an agent who is risk neutral in the standard notion, $u''(\cdot) = 0$, but exhibits loss aversion $\lambda > 1$.

If the principal is restricted to offering deterministic contracts with two possible signals $s \in \{1, 2\}$, the deterministic contract takes the form of a bonus contract: the agent is paid a base wage $\underline{w}$ if the bad signal is realized and is paid the base wage $\underline{w}$ plus a bonus $b > 0$ if the good signal is realized.

Under this deterministic contract, the agent prefers the high action a_H over the low action a_L if his utility from the high action exceeds his utility from the low action. This is the case if and only if

$$\underline{w} + q_H b - (\lambda - 1) q_H (1 - q_H) b - c \geq \underline{w} + q_L b - (\lambda - 1) q_L (1 - q_L) b$$

$$\text{(IC-D)} \quad \Leftrightarrow \; (q_H - q_L) b - (\lambda - 1) \left[q_H (1 - q_H) - q_L (1 - q_L) \right] b \geq c$$

Because both the participation and incentive constraints are binding, the principal's cost minimization problem is equivalent to minimizing the agent's loss premium conditional on a_H subject to the incentive constraint. I examine whether there exists a stochastic contract that satisfies the incentive constraint and at the same time reduces the loss premium that the principal has to pay.

Assuming that the principal can employ stochastic contracts, I consider the stochastic contract that takes the turning-a-blind-eye structure: the principal pays a high wage with probability 1 if she observes the good signal, while if she observes the bad signal she stochastically ignores it by paying the high wage with probability p_1 and paying a low wage with probability $1 - p_1$. It follows directly from Lemma 1 that this stochastic contract satisfies the incentive constraint and implements the high action. I examine whether the stochastic contract benefits the principal from a cost perspective in the following proposition.

PROPOSITION 1: *Suppose $u''(\cdot) = 0$ and $\lambda - 1 > \frac{1-q_H}{1-q_L}$. Then, there exists a stochastic contract with the wage structure $w_1 < w_2 = w_3 = w_4$ that strictly dominates the optimal deterministic contract.*

Besides remedying the implementation problem, the stochastic contract benefits the principal from a cost perspective: the principal pays a lower loss premium to the agent in the stochastic contract. Intuitively, the agent's loss premium depends on two variables: (i) the bonus size b and (ii) the probability with which the agent feels a loss when a deviation from his reference point occurs $q_H(1 - q_H)$, which following Herweg, Müller and Weinschenk (2010) I refer to as "loss probability". The loss probability is an inverted U-shaped function; it reaches its maximum when getting a bonus is completely random, that is, $q_H = 1/2$, and it reaches its minimum of zero as the bonus probability moves to the extremes, that is, $q_H = 0$ or $q_H = 1$. By employing the stochastic contract that pays the low wage only if the worst outcome ($i = 1$) is realized and pays the high wage for all other outcomes, the principal increases the bonus probability closer to one and thereby reduces the associated loss probability closer to zero.

Although the stochastic contract decreases the probability that the agent feels a loss, it increases the bonus size b required to incentivize the agent to work. As the probability of getting a bonus increases, the outcome distribution under the high action resembles that under the low action. Thus, to satisfy the incentive constraint, the principal needs a higher bonus. Put together, the stochastic contract has two opposing effects on the loss probability and the bonus size. Although the insurance against wage uncertainty may come at the cost of a larger bonus required to induce the agent to work, the positive effect of the reduced loss probability outweighs the negative effect of the increased bonus size if the agent is sufficiently loss averse.

Figure 2 illustrates the dominance of the stochastic contract for a simple example with $q_H = 0.8, q_L = 0.3$, and $p_1 = 0.75$. The dashed line in Figure 2 shows the principal's implementation cost under the optimal deterministic contract, and the solid line shows the minimum cost under the stochastic contract with the wage structure $w_1 < w_2 = w_3 = w_4$. Given $q_H = 0.8$ and $q_L = 0.3$, the condition $\lambda - 1 > \frac{1-q_H}{1-q_L}$ in Proposition 1 translates to $\lambda > 1.29$. As shown in Figure 2, for $\lambda \in [1, 1.29]$, the optimal deterministic contract yields a lower cost for the principal, whereas for $\lambda > 1.29$, the stochastic contract strictly

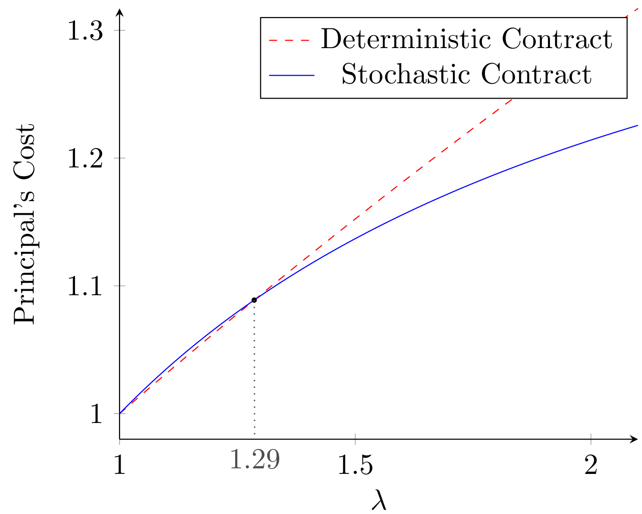

Figure 2. : Principal's cost under stochastic contracts vs. deterministic contracts

Note: The figure illustrates the principal's cost under stochastic contracts and deterministic contracts for $q_H = 0.8, q_L = 0.3$, $p_1 = 0.75$, $p_2 = 1$, and $c = 1$. The dashed line shows the principal's implementation cost in the optimal deterministic contract. The solid line shows the principal's minimum cost in the stochastic contract with the wage structure $w_1 < w_2 = w_3 = w_4$.

dominates the optimal deterministic contract. The higher the degree of loss aversion, the larger the relative benefit of using the stochastic contract for the principal.

Surprisingly, the condition on the degree of loss aversion in Proposition 1 is much weaker than that previously established in the literature. Herweg, Müller and Weinschenk (2010) establish that turning a blind eye enables the principal to achieve a lower cost when $\lambda > 2$.[7] Notice that in Proposition 1 the condition $\frac{1-q_H}{1-q_L} + 1$ is strictly smaller than 2; this would imply a larger set of degrees of loss aversion than previously thought under which stochastic contracts strictly dominate deterministic contracts.

A second interesting observation is that as the performance signals become more informative about the agent's action, the principal favors the stochastic contract under a wider range of degrees of loss aversion. Let us consider two extreme cases. If the signals are highly uninformative, that is, $\frac{1-q_H}{1-q_L} \to 1$, then the most restrictive condition under which the stochastic contract dominates deterministic contracts becomes $\lambda > 2$, which coincides with the well-established condition in

[7]See Proposition 7 in Herweg, Müller and Weinschenk (2010).

the literature. The condition on the degree of loss aversion, however, gets weaker as the performance signals provide more information about the agent's action. At the other extreme, if the signals are highly informative, that is, $\frac{1-q_H}{1-q_L} \to 0$, then the condition becomes $\lambda > 1$. This means that if the signals provide almost precise information about the agent's action, then the principal benefits from using the stochastic contract almost all the time. The logic is that when the given signals are very informative, the principal provides further wage certainty at a negligible cost and prefers to do so to a large extent. Put differently, in the limit, the stochastic contract strictly dominates deterministic contracts for almost any degree of loss aversion.

B. *Non-Existence of The Second-Best Optimal Contract*

I now focus on the cases where stochastic contracts strictly dominate deterministic contracts. Formally, I assume that $u''(\cdot) = 0$ and $\lambda - 1 > \frac{1-q_H}{1-q_L}$. In this part, I attempt to characterize the second-best optimal stochastic contract, assuming for now that the solution exists.

Similar to the finding by Herweg, Müller and Weinschenk (2010), a first important observation is that the optimal stochastic contract should take the form of a bonus contract. When an agent is risk neutral but loss averse, it is optimal for the principal to pool as many informative signals as possible into a bonus set and pay a high wage only if the realized signal lies in this bonus set. The logic is that when facing the risk-neutral agent, the principal cannot capitalize on a higher degree of wage differentiation. On the other hand, pooling wages together helps the loss-averse agent avoid unfavorable comparisons and yields him a higher expected utility. To satisfy the incentive constraint, the optimal contract requires a minimum degree of wage differentiation in that the principal offers two wage levels—a base wage and a bonus—no matter how rich the signal space is.

It remains to determine which outcomes $i \in \{1, \ldots, 4\}$ should be included in the bonus set. Given any contract $(\hat{w}_i)_{i=1}^4$ that the principal offers, I can relabel the outcomes i such that this contract is equivalent to a contract $(w_i)_{i=1}^4$ of a (weakly) increasing wage profile with $w_{i-1} \le w_i$ for all $i \in \{2, 3, 4\}$. Thus the bonus set can be one of three options: (i) the bonus set includes only the highest outcome $\{w_4\}$, (ii) the bonus set includes the two highest outcomes $\{w_4, w_3\}$, or (iii) the bonus set includes all but the lowest outcome $\{w_4, w_3, w_2\}$. I examine the option (i) in the following lemma.

LEMMA 2: *Suppose $u''(\cdot) = 0$ and $\lambda > 1$. Then, any stochastic contract with the wage structure $w_1 = w_2 = w_3 < w_4$ is weakly dominated by the optimal deterministic contract.*

Lemma 2 states that any stochastic contract that adds noise to the good signal such that only the highest outcome $i = 4$ receives a bonus is weakly dominated by the optimal deterministic contract. In other words, the bonus set that includes only the highest outcome $\{w_4\}$ is weakly dominated by the bonus set that includes the two highest outcomes $\{w_4, w_3\}$.

Intuitively, a stochastic contract that rewards only the highest outcome reduces the probability of getting a bonus; a slim chance of getting a bonus in turn decreases the agent's expected net loss, but much more when the agent shirks than when he works. Because the agent cares about minimizing the expected loss, this implies that the stochastic contract of the wage structure $w_1 = w_2 = w_3 < w_4$ makes shirking more attractive and worsens the implementation problem under loss aversion. Moreover, the principal requires a substantially higher bonus to motivate the agent to work. Due to the worsening implementation problem, the negative effect of an increased bonus outweighs the positive effect of a reduced loss probability, consequently the principal's implementation cost actually increases with such a stochastic contract.

Thus, option (i)—the bonus set that includes only the highest outcome $\{w_4\}$—cannot be the optimal bonus set. I now examine the option (ii)—the bonus set that includes the two highest outcomes $\{w_4, w_3\}$. Note that the option (ii) coincides with the deterministic contract. As in Proposition 1, the optimal deterministic contract is strictly dominated by the stochastic contract with the wage structure $w_1 < w_2 = w_3 = w_4$. Taking these two observations together, it is thus optimal to include all but the worst outcome in the bonus set.

With the bonus set including all except for the worst outcome $i = 1$, I derive the principal's implementation cost for a given stochastic structure. The comparative statics of the principal's implementation cost with respect to the probability of getting a bonus p_1 reveal an insight about the existence of the second-best optimal stochastic contract, which is covered in the following proposition.

PROPOSITION 2: *Suppose $u''(\cdot) = 0$ and $\lambda - 1 > \frac{1-q_H}{1-q_L}$. Then, the second-best optimal stochastic contract does not exist.*

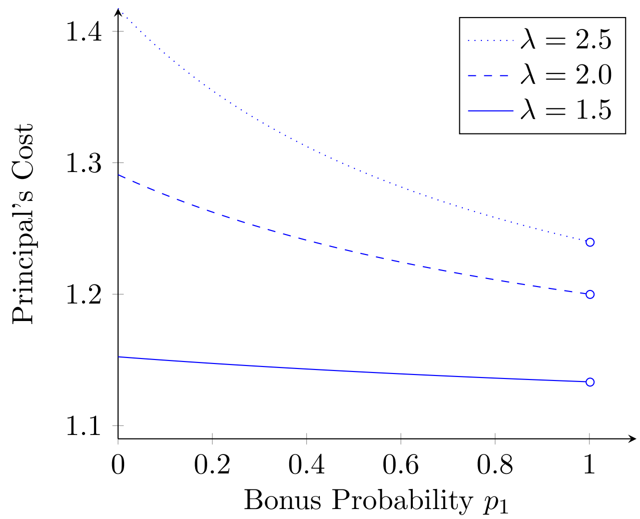

Figure 3. : Principal's cost as a function of the bonus probability

Note: The figure illustrates the principal's cost under the stochastic contract of the wage structure $w_1 < w_2 = w_3 = w_4$ for $q_H = 0.8, q_L = 0.3$, $p_2 = 1$, and $c = 1$.

The solution to the principal's problem with the above stochastic contract is not well-defined. The reason is that the principal can always achieve a lower cost by further increasing the probability of getting a bonus p_1 close to one and rendering the penalty harsher in the event of the worst outcome. However, p_1 cannot reach the value of one, as the contract then becomes a fixed wage contract that does not satisfy the incentive constraint. In the limit, the principal's cost C_r in the stochastic contract is given by

$$\lim_{p_1 \to 1} C_r = c + \frac{\lambda - 1}{\lambda} \cdot \frac{(1 - q_H)c}{q_H - q_L}$$

Figure 3 illustrates how the principal's implementation cost changes with respect to the probability of getting a bonus p_1 for a simple example with $q_H = 0.8$ and $q_L = 0.3$. The solid, dashed, and dotted lines correspond to the principal's cost under $\lambda = 1.5$, $\lambda = 2$, and $\lambda = 2.5$ respectively. All the lines exhibit a downward trend, implying that the principal's cost decreases as p_1 increases. However, there is a discontinuity, depicted as empty circles, at $p_1 = 1$. If $p_1 = 1$, the principal cannot induce the agent to work, and her implementation cost becomes prohibitively high.

C. Limited Liability

The non-existence of the second-best optimal stochastic contract hinges on the principal's desire to insure the agent against wage uncertainty to the largest possible extent, and thereby to further reduce her cost, if the agent is sufficiently loss averse. On the other hand, to motivate the agent to work in the face of such insurance, the principal punishes the agent very harshly when the worst outcome is realized. If the punishment for the worst outcome is, however, limited, the principal faces an upper bound of how much wage certainty she can provide to the agent. In this part, I show that the second-best optimal stochastic contract exists if the principal faces a limited liability constraint, and I characterize the second-best optimal contract.

Analogous to the previous analysis, it can be shown that the optimal bonus set consists of all but the worst outcome. I thus restrict my attention to stochastic contracts of the wage structure $w_1 < w_2 = w_3 = w_4$. Let f_H and f_L be the probability of getting a bonus conditional on the agent's high and low actions, respectively, that is, $f_H = q_H + p_1(1-q_H)$ and $f_L = q_L + p_1(1-q_L)$. The principal's problem becomes

$$\min_{\underline{w},b,p_1} \ \underline{w} + f_H b$$

subject to

$$\text{(PC)} \qquad \underline{w} + f_H b - (\lambda-1) b f_H (1-f_H) \geq c$$

$$\text{(IC)} \qquad b(f_H - f_L) - (\lambda-1) b [f_H(1-f_H) - f_L(1-f_L)] \geq c$$

$$\text{(LL)} \qquad \underline{w} \geq 0$$

Because the IC binds at optimum (else, the principal can reduce b and increase $\underline{w}$ by a small amount such that PC remains unchanged), the optimal bonus size can be written as a function of p_1:

$$b^*(p_1) = \frac{c}{(q_H - q_L)(1-p_1)[1-(\lambda-1)(1-q_H-q_L-p_1(2-q_H-q_L))]}$$

The LL constraint is also binding at optimum. Else, by reducing $\underline{w}$ by a small amount, the principal decreases the expected payment without changing IC or violating LL. Thus, the principal's cost in the stochastic contract is given by $C_r(p_1) = f_H b^*$. The principal reduces her implementation cost by using the stochastic contract if and only if the following assumption holds.

ASSUMPTION 1 (A1): $\frac{1}{\lambda - 1} < \max\{(2 - q_H - q_L)(1 + q_H) - 1, 1 - q_L\}$

Assumption (A1) requires that the degree of loss aversion is sufficiently large. The first term in the bracket corresponds to a sufficient and necessary condition for the principal's minimum cost $C_r(p_1)$ to be locally decreasing at $p_1 = 0$. The second term in the bracket ensures that there exists $p_1 \in (0, 1)$ such that the PC is satisfied. Given that Assumption (A1) holds, there exists a stochastic contract that strictly dominates the optimal deterministic contract under limited liability.[8] Solving for the optimal p_1^* that minimizes $C_r(p_1)$, I characterize the second-best optimal stochastic contract in the following proposition.

PROPOSITION 3: *Suppose (A1) holds, $u''(\cdot) = 0$, and $w \geq 0$. Then, the second-best optimal stochastic contract exists. The optimal stochastic contract pays $b^*(p_1^*)$ with a probability of one when the good signal is realized and with a probability of p_1^* when the bad signal is realized. The optimal $p_1^* \in (0, 1)$ is given by*

$$p_1^* = \max\{p', p''\}$$

where $p' = \frac{1}{1-q_H}\left(\sqrt{1 - \frac{\lambda}{\lambda-1} \cdot \frac{1-q_H}{2-q_H-q_L}} - q_H\right)$ *and* $p'' = 1 - \frac{1}{(\lambda-1)(1-q_L)}$

Figure 4 illustrates the second-best optimal stochastic contract under limited liability with a simple example of $q_H = 0.8$, $q_L = 0.3$, and $\lambda = 3$. With this parameter specification, the principal can implement the desired action with a deterministic contract that reaches the lowest cost of $C_d^* = 1.53$. Assumption (A1), translating to $\lambda > 2.43$, is satisfied under the specification of $\lambda = 3$. The participation constraint is satisfied whenever the bonus probability $p_1 \geq p''$, and the principal's cost under the stochastic contract reaches its minimum at $p_1 = p'$. In this example, at the bonus probability $p' = 0.08$, which falls below $p'' = 0.29$, the participation constraint is violated. Thus, the optimal bonus probability $p_1^* = \max\{p', p''\} = 0.29$. The second-best optimal stochastic contract pays $b^*(p_1^*) = 1.63$ with a probability of one if the principal observes the good signal $s = 2$ and

[8]This condition is consistent with Daido and Murooka's (2013) Proposition 3. Similar to their proposition, I consider when stochastic compensation is optimal under limited liability. Extending their insights, I additionally impose the participation constraint and identify a range of loss aversion under which stochastic contracts strictly dominate the optimal deterministic contract.

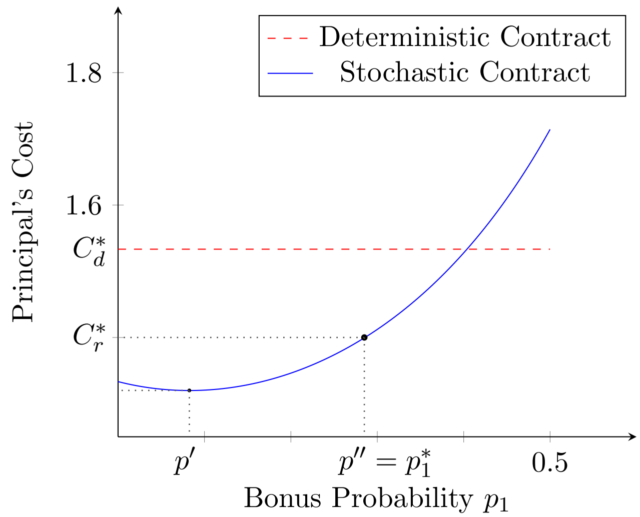

Figure 4. : Principal's cost under limited liability

Note: The figure illustrates the principal's cost under stochastic contracts and deterministic contracts under limited liability for $q_H = 0.8, q_L = 0.3$, $p_2 = 1$, $c = 1$, and $\lambda = 3$. The dashed line shows the principal's implementation cost under the optimal deterministic contract. The solid line shows the principal's minimum cost under the stochastic contract with the wage structure $w_1 < w_2 = w_3 = w_4$.

with a probability of $p_1^* = 0.29$ if she observes the bad signal $s = 1$. Thus, the principal obtains the optimal cost of $C_r^* = 1.40$, which is strictly lower than C_d^*.

If the agent is subject to limited liability, the solution of the principal's problem is well-defined. Intuitively, limited liability limits the extent to which the principal can punish the agent in the event of bad outcomes and in turn her ability to insure the agent against wage uncertainty. Put differently, the principal does not benefit from increasing the bonus probability p_1 close to 1 under the limited liability constraint. As the base wage $\underline{w}$ is bounded by zero, in order to motivate the agent to work, the bonus b becomes substantially large after a certain level of wage certainty.

V. Discussion

A. Finite Signals

The main analysis focuses on a simple case of two signals. In this subsection, I examine whether the main result—the principal benefits from stochastically ignoring bad signals—is robust under the assumption of finite signals. While one can argue that observing finitely many signals helps the principal fine-tune the signals to wage payments, I will show that the strict dominance of stochastic contracts holds in the setting of finite signals.

Suppose that the principal observe finite signals $s \in S = \{1, \ldots, N\}$. Denote q_s^H and q_s^L the probability of observing signal s conditional on the agent's high and low action, respectively. Under stochastic contracts, the principal can add a lottery $(p_s, 1-p_s)$ after observing signal s. For technical convenience, I make the following standard assumption.

ASSUMPTION 2 (A2): *For all $s \in S$,*
(i) $q_s^H, q_s^L \in (0,1)$ (full support),
(ii) $\frac{q_s^H}{q_s^L}$ is increasing in s (monotone likelihood ratio property).

Assumption (A2, i) ensures that all signals occur with positive probability for all action $a \in \{a_H, a_L\}$, and Assumption (A2, ii) implies that signals can be ranked according to their likelihood ratios such that the higher the realized signal s, the more likely that the agent works. The robustness of the main result under finite signals is presented in the following Proposition.

PROPOSITION 4: *Consider finite signals $s \in \{1, \ldots, N\}$. Suppose (A2) holds, $u''(\cdot) = 0$ and $\lambda - 1 > \frac{1-q_H^*}{1-q_L^*}$, where q_H^* and q_L^* are the probability of getting the high wage conditional on a_H and a_L under the optimal deterministic contract. Then, there exists a stochastic contract that strictly dominates the optimal deterministic contract.*

Intuitively, the optimal deterministic contract partitions the signal space into a set of bad signals $s \in S^- = \{1, \ldots, k\}$ and a set of good signals $s \in S^+ = \{k+1, \ldots, N\}$. Consider a stochastic contract that turns a blind eye on bad signals: it pays a high wage for all $s \in S^+ = \{k+1, \ldots, N\}$ and it adds a lottery that pays the same high wage with a positive probability p_1 if a bad signal $s \in S^- = \{1, \ldots, k\}$ is realized. This stochastic contract helps reduce the principal's cost by reducing the probability that agent feels a loss.

B. Loss Aversion and Risk Aversion

So far, it has been assumed that the agent is purely loss averse and risk neutral. This subsection discusses a general case when the agent is loss averse and risk averse and examines whether the main finding of Proposition 1 holds under the general case.

The combination of risk and loss aversion creates a trade-off for the principal. On the one hand, risk aversion imposes an additional cost for the principal to implement a stochastic contract, because the risk-averse agent dislikes noise. On the other hand, loss aversion implies that the principal still benefits from a stochastic contract as it helps reduce the loss probability and the associated loss premium.

The principal prefers a stochastic contract if the benefit of implementing a stochastic contract outweighs the cost. More precisely, I will argue that given any degree of relative risk aversion (i.e., at any given cost of implementing a stochastic contract), if the agent is sufficiently loss averse (i.e., if the benefit is sufficiently large), then the principal strictly prefers a stochastic contract over any deterministic contracts.

Suppose the agent exhibits a constant relative risk aversion (CRRA) consumption utility, which is given by $u(w) = \frac{w^{1-\pi}-1}{1-\pi}$ with $0 < \pi < 1$. Thus, the parameter π measures the agent's degree of relative risk aversion. The following Proposition shows that the main finding on the dominance of the stochastic contract is robust for the general case of loss aversion and risk aversion.

PROPOSITION 5: *Suppose* $u''(\cdot) < 0$ *and* $\lambda > 1$. *Consider two actions and two signals. For any degree of relative risk aversion* π, *there exists a sufficiently large* λ *such that a stochastic contract with the wage structure* $w_1 < w_2 = w_3 = w_4$ *strictly dominates the optimal deterministic contract.*

Put differently, if loss aversion is more important than risk aversion in the agent's preferences, then the principal puts more weight on reducing the loss premium than on reducing risk premium, and a stochastic contract with the wage structure $w_1 < w_2 = w_3 = w_4$ strictly dominates any deterministic contracts. The principal prefers to add noise in the contract to reduce the loss probability and the corresponding loss premium even at the cost of a higher risk premium.

C. Alternative Notions of Loss Aversion

The notion of loss aversion crucially depends on how the reference point is conceptualized. In my model, the CPE reference point has two important features. First, it allows for stochastic reference points: the agent compares a realized outcome with all possible outcomes. This pairwise comparison implies a possibility of "mixed feeling", that is, the same realized outcome can be perceived as both a gain and a loss at the same time, depending on which possible outcomes the agent expects. Second, the reference point is formed after the decision is made and hence is influenced by the chosen decision. Thus, the reference point is endogenously determined by recent expectations.

A notion related to the CPE concept is the forward-looking disappointment aversion according to Bell (1985), Loomes and Sugden (1986), or Gul (1991). Under the disappointment aversion model, the reference point is also formed after the decision is made, but the reference point takes the form of the certainty equivalent of the prospect, and hence it admits only static reference points. The certainty equivalent of the prospect is a point estimate and does not allow for mixed feelings; the agent feels a gain if the realized outcome is above it and vice versa. As it turns out, even in this case, stochastic contracts help the principal reduce the implementation cost beyond what is achieved under deterministic contracts.[9]

PROPOSITION 6: *Suppose the agent exhibits disappointment aversion according to Bell (1985),* $u''(\cdot) = 0$*, and* $\lambda - 1 > \frac{1-q_H}{1-q_L}$*. Consider two actions and two signals. Then, there exists a stochastic contract with the wage structure* $w_1 < w_2 = w_3 = w_4$ *that strictly dominates the optimal deterministic contract.*

The forward-looking disappointment according to Bell (1985) implies that the agent first calculates an expected outcome and then compares the realized outcome with his expectation. Under a deterministic contract, if a bonus is realized, the agent feels elated as the realized outcome is better than the expected one. However, if a bonus is not realized, the agent instead feels disappointed as the realized outcome is worse than the expected one. By increasing the bonus probability in the stochastic contract, the principal simultaneously increases the probability that the agent feels elated and reduces the probability that he feels disappointed. Because the agent prioritizes minimizing the feeling of disappointment, if he is sufficiently disappointment averse, the principal can capitalize on the stochastic contract to reduce her implementation cost.

An alternative specification of the reference point is that it is given exogenously and does not internalize the effect of the decision, namely the *preferred personal*

[9]De Meza and Webb (2007) examine the concept of Gul (1991), which is closely related to Bell (1985), and find that the optimal contracts have intermediate intervals in which wages are insensitive to performance.

equilibrium (PPE). In PPE, the agent can choose his optimal action only from the actions he knows he will follow through, whereas in CPE he can commit to the action. The analysis of the optimal contract is very similar and gives rise to a similar result. However, it is known that the distaste for risk is stronger when the decision is made up front, as in CPE, than when the decision is made later, as in PPE. The principal benefits from stochastic contracts that insure the agent against wage uncertainty to a lesser extent.

PROPOSITION 7: *Suppose the agent exhibits the PPE loss aversion,* $u''(\cdot) = 0$, $q_H + 2q_L \leq 2$, *and* $\lambda - 1 > \frac{1-q_H}{1-q_L}$. *Consider two actions and two signals. Then, there exists a stochastic contract with the wage structure* $w_1 < w_2 = w_3 = w_4$ *that strictly dominates the optimal deterministic contract.*

The robustness of the dominance of stochastic contracts suggests that noise should be generally added to performance measures in the optimal contract for loss averse agents. Put differently, loss aversion implies a first-order aversion to wage uncertainty, and this creates incentives for the principal to insure the agent against this uncertainty. By employing stochastic contracts, the principal manipulates the outcome distribution in her favor and provides higher wage certainty to the agent. When loss aversion plays a role, the principal capitalizes on this reduction in uncertainty and achieves a lower cost.

VI. Conclusion

This article studies the optimal contract design under moral hazard and loss aversion and finds that the optimal contract adds noise in the event of bad outcomes to insure the loss-averse agent against wage uncertainty. To reach this finding, I modify the standard moral hazard model with two departures: the agent is expectation-based loss averse, and the principal can add noise in the contract to manipulate the outcome distribution in her favor. Importantly, the principal fully controls where to add noise and how to structure noise in the contract, that is, the structure of stochastic contracts.

There are two key takeaways from this article. First, the principal is strictly better off with stochastic contracts, as compared to deterministic contracts, in implementing the desired action if the agent is sufficiently loss averse. This result relates to the literature on behavioral contract theory, which has pointed out that if deterministic contracts face an implementation problem, turning a blind eye (Herweg, Müller and Weinschenk, 2010) or team incentives (Daido and Murooka, 2016) help the principal induce the agent to work. Contributing to this strand of literature, I find that even if deterministic contracts do not face the implementation problem, the principal can still reduce her cost by employing stochastic contracts. In fact, if the signals are highly informative about the agent's action, stochastic contracts strictly dominate deterministic contracts for almost any degree of loss aversion. The dominance of stochastic contracts is also robust towards introducing risk aversion in the agent's preferences. Thus, this finding has an important implication for designing contracts for loss-averse agents: the principal has an incentive to add noise after the bad signal is realized to insure the agent against wage uncertainty.

Second, limited liability mitigates the non-existence problem of the second-best optimal stochastic contract. Instead of the implementation problem, stochastic contracts face the non-existence problem that the optimal contract does not exist, because the principal has an incentive to insure the agent to the largest possible extent. Given a wide range of degrees of loss aversion under which stochastic contracts dominate deterministic contracts, the non-existence problem proves to be severe. To solve the non-existence problem, I find that limited liability helps restore the existence of the second-best optimal contract. This finding highlights the importance of limited liability in stochastic contracts to ensure that the second-best optimal contract exists.

Given that loss aversion is an important behavioral trait, this article helps explain the relevance of stochastic contracts (e.g., dismissal contracts) in the real world. Further research can help shed light on the interaction of loss aversion with other behavioral or cognitive biases, such as overconfidence, that may induce the employee to have an incorrect model of the world. Understanding the interaction of these behavioral biases and their implications for the optimal contract design are crucial for management more broadly.

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Mathematical Appendix

Proofs of Propositions and Lemmas

PROOF OF LEMMA 1:

Suppose $u''(\cdot) \leq 0$ and $\lambda \geq 1$. For every λ, there exists a stochastic contract such that the action a_H can be implemented.

Without loss of generality, assume $1 > p_1 \geq 1/2$. Consider a contract of the form

$$u_i = \begin{cases} \underline{u} + b & \text{for } i > 1 \\ \underline{u} & \text{for } i = 1 \end{cases}$$

where $b > 0$.

Let f_1^H and f_1^L be the probability of getting a bonus conditional on the agent's high and low action respectively, i.e., $f_1^H = P[i > 1|a_H] = q_H + p_1(1 - q_H)$ and $f_1^L = P[i > 1|a_L] = q_L + p_1(1 - q_L)$. Under this contractual form, (IC) is given by

$$\text{(IC)} \qquad b(f_1^H - f_1^L) - (\lambda - 1)b[f_1^H(1 - f_1^H) - f_1^L(1 - f_1^L)] = c$$

which can be rewritten as

$$\text{(IC')} \qquad b\{(f_1^H - f_1^L)[1 - (\lambda - 1)(1 - f_1^H - f_1^L)]\} = c$$

Under this stochastic contract, $f_1^H = q_H + p_1(1 - q_H)$ and $f_1^L = q_L + p_1(1 - q_L)$. It is straight-forward to see that $f_1^H > f_1^L$ as $q_H > q_L$ and $p_1 < 1$.
Consider

$$\begin{aligned} 1 - f_1^H - f_1^L &= 1 - (q_H + p_1(1 - q_H)) - (q_L + p_1(1 - q_L)) \\ &= 1 - q_H - q_L - p_1(2 - q_H - q_L) \end{aligned}$$

Notice for $p_1 \geq 1/2$, this above term is strictly negative. This implies the term in curly brackets in (IC') is strictly positive for $1 > p_1 \geq 1/2$. Hence, with $c > 0$, b can always be chosen such that (IC) is met.
The binding participation constraint can be written as follows

$$\underline{u} + bf_1^H - (\lambda - 1)bf_1^H(1 - f_1^H) = c$$

(PC) is satisfied whenever $\underline{u}$ is chosen as above.

PROOF OF PROPOSITION 1:

Suppose $u''(\cdot) = 0$ *and* $\lambda - 1 > \frac{1-q_H}{1-q_L}$. *Then, there exists a stochastic contract with the wage structure* $w_1 < w_2 = w_3 = w_4$ *that strictly dominates the optimal deterministic contract.*

Assume $1 > p_1 \geq 1/2$. Consider a stochastic contract of the form

$$w_i = \begin{cases} \underline{w} + b & \text{for } i > 1 \\ \underline{w} & \text{for } i = 1 \end{cases}$$

where $b > 0$. The non-emptiness of the constraint set follows from Lemma 1.

Let f_1^H and f_1^L be the probability of getting a bonus conditional on the agent's high and low action respectively, i.e., $f_1^H = P[i > 1|a_H] = q_H + p_1(1 - q_H)$ and $f_1^L = P[i > 1|a_L] = q_L + p_1(1 - q_L)$.
Consider any $p_1 \in [\frac{1}{2}, 1)$. The principal's problem becomes

$$\min_{\underline{w},b} \ \underline{w} + f_1^H b$$

subject to

$$\text{(PC)} \qquad \underline{w} + f_1^H b - (\lambda - 1)bf_1^H(1 - f_1^H) = c$$

$$\text{(IC)} \qquad b(f_1^H - f_1^L) - (\lambda - 1)b[f_1^H(1 - f_1^H) - f_1^L(1 - f_1^L)] = c$$

From (IC), the optimal bonus size is given by

$$b = \frac{c}{(f_1^H - f_1^L)[1 - (\lambda - 1)(1 - f_1^H - f_1^L)]}$$

Recall that $f_1^H = q_H + p_1(1 - q_H)$ and $f_1^L = q_L + p_1(1 - q_L)$. Under the stochastic contract of this form, the principal's cost, $C_r = c + (\lambda - 1)f_1^H(1 - f_1^H)b$, is given by

$$C_r = c + \frac{(\lambda - 1)[q_H + p_1(1 - q_H)](1 - q_H)(1 - p_1)c}{(q_H - q_L)(1 - p_1)[1 - (\lambda - 1)(1 - q_H - q_L - p_1(2 - q_H - q_L))]}$$

Suppose that the optimal deterministic contract exists.[10] Then the principal's

[10]If the principal's constraint set is empty under deterministic contracts, then it is assumed that the

cost under the optimal deterministic contract (i.e., $p_1 = 0$) is given by

$$C_d = c + \frac{(\lambda-1)q_H(1-q_H)c}{(q_H-q_L)[1-(\lambda-1)(1-q_H-q_L)]}$$

The stochastic contract reduces the principal's cost if and only if $C_d \geq C_r$.

$$\Leftrightarrow \frac{q_H}{1-(\lambda-1)(1-q_H-q_L)} \geq \frac{q_H + p_1(1-q_H)}{1-(\lambda-1)(1-q_H-q_L-p_1(2-q_H-q_L))}$$

Because the solution exists for both deterministic and stochastic contracts, both denominators are positive. Cross multiply the inequalities.
Notice the term $q_H[1-(\lambda-1)(1-q_H-q_L)]$ is present on both sides. The inequality is reduced to

$$\Leftrightarrow q_H(\lambda-1)p_1(2-q_H-q_L) \geq p_1(1-q_H)[1-(\lambda-1)(1-q_H-q_L)]$$
$$\Leftrightarrow q_H(\lambda-1)(2-q_H-q_L) \geq (1-q_H)[1-(\lambda-1)(1-q_H-q_L)]$$

Removing the term $q_H(\lambda-1)(1-q_H-q_L)$ on both sides, I have

$$\Leftrightarrow q_H(\lambda-1) \geq 1-(\lambda-1)(1-q_H-q_L) - q_H$$
$$\Leftrightarrow 0 \geq 1-q_H-(\lambda-1)(1-q_L)$$
$$\Leftrightarrow \lambda - 1 \geq \frac{1-q_H}{1-q_L}$$

Because $\lambda - 1 > \frac{1-q_H}{1-q_L}$, $C_r < C_d$. This completes the proof.

PROOF OF LEMMA 2:

Suppose $u''(\cdot) = 0$ and $\lambda \geq 1$. Then, any stochastic contract with the wage structure $w_1 = w_2 = w_3 < w_4$ is weakly dominated by the optimal deterministic contract.

Consider a stochastic contract of the form

$$w_i = \begin{cases} \underline{w} + b & \text{for } i = 4 \\ \underline{w} & \text{for } i < 4 \end{cases}$$

principal's cost becomes prohibitively high. It follows directly that stochastic contracts, which enable the principal to implement the desired action, strictly dominate deterministic contracts.

where $b > 0$. Let f_4^H and f_4^L be the probability of getting a bonus conditional on the agent's high and low action respectively, i.e., $f_4^H = P[i = 4|a_H] = p_2 q_H$ and $f_4^L = P[i = 4|a_L] = p_2 q_L$.
The principal's problem becomes

$$\min_{\underline{w}, b} \ \underline{w} + f_4^H b$$

subject to

$$\text{(PC)} \qquad \underline{w} + f_4^H b - (\lambda - 1) b f_4^H (1 - f_4^H) = c$$

$$\text{(IC)} \qquad b(f_4^H - f_4^L) - (\lambda - 1) b [f_4^H (1 - f_4^H) - f_4^L (1 - f_4^L)] = c$$

Suppose that the above constraint set is non-empty, the optimal bonus size is given by

$$b = \frac{c}{(f_4^H - f_4^L)[1 - (\lambda - 1)(1 - f_4^H - f_4^L)]}$$

Recall that $f_4^H = p_2 q_H$ and $f_4^L = p_2 q_L$. Under the stochastic contract of this form, the principal's cost, $C = c + (\lambda - 1) f_4^H (1 - f_4^H) b$, is given by

$$C = c + \frac{(\lambda - 1) p_2 q_H (1 - p_2 q_H) c}{p_2 (q_H - q_L)[1 - (\lambda - 1)(1 - p_2 q_H - p_2 q_L)]}$$

Note that if the constraint set for the above stochastic contract is non-empty, then the constraint set for the deterministic contract is also non-empty. Thus, the principal's cost under the optimal deterministic contract (i.e., $p_2 = 1$) is given by

$$C_d = c + \frac{(\lambda - 1) q_H (1 - q_H) c}{(q_H - q_L)[1 - (\lambda - 1)(1 - q_H - q_L)]}$$

It is straight-forward to see that $C \geq C_d$ for any $1 \geq p_2 > 0$, because $1 - (\lambda - 1)(1 - q_H - q_L) \geq 1 - (\lambda - 1)(1 - p_2 q_H - p_2 q_L)$ and $1 - p_2 q_H \geq 1 - q_H$.

PROOF OF PROPOSITION 2:

Suppose $u''(\cdot) = 0$ and $\lambda - 1 > \frac{1 - q_H}{1 - q_L}$. Then, the second-best optimal stochastic contract does not exist.

Suppose, by contradiction, the solution for the principal's problem exists.

I decompose the principal's problem into two subproblems. First, for a given stochastic structure (p_1, p_2), I derive the optimal wage payments that implement a_H. Second, I choose the stochastic structure to achieve the lowest cost.

Step 1: Given any contract $(\hat{w}_i)_{i=1}^4$ the principal offers, I can relabel the states such that this contract is equivalent to a contract $(w_i)_{i=1}^4$ of an (weakly) increasing wage profile with $w_{i-1} \leq w_i$ for all $i \in \{2,3,4\}$. Let $b_i = w_i - w_{i-1} \geq 0$ for all $i \in \{2,3,4\}$. Let f_i^H and f_i^L be the probability that state i is realized conditional on a_H and a_L respectively.

The principal's problem can be rewritten as

$$\min_{b_2,\ldots,b_4} (\lambda - 1) \sum_{i=2}^{4} b_i \sum_{\tau=i}^{4} f_\tau^H \sum_{t=1}^{i-1} f_t^H$$

subject to

$$\text{(IC)} \qquad \sum_{i=2}^{4} b_i \beta_i = c$$

$$b_i \geq 0 \ \ \forall i \in \{2,3,4\}$$

where

$$\beta_i := \left(\sum_{\tau=i}^{4} (f_\tau^H - f_\tau^L) \right) - (\lambda - 1) \left(\sum_{\tau=i}^{4} f_\tau^H \sum_{t=1}^{i-1} f_t^H - \sum_{\tau=i}^{4} f_\tau^L \sum_{t=1}^{i-1} f_t^L \right)$$

The principal's problem is a linear programming problem. It is well known that if a linear programming has a solution, this (unique) solution is an extreme point of the constraint set. All extreme points of the constraint set are characterised by the following property: $b_i > 0$ for exactly one state $i \in \{2,3,4\}$ and $b_t = 0$ for all $t \neq i, t \in \{2,3,4\}$.

It remains to determine for which state $i \in \{2,3,4\}$ the bonus is set strictly positive. From Lemma 2 and Proposition 1 if $\lambda - 1 > \frac{1-q_H}{1-q_L}$, the second-best optimal stochastic contract has the optimal wage structure $w_1 < w_2 = w_3 = w_4$.

Step 2: I now consider the optimal stochastic structure p_1 to achieve the lowest cost. Recall that under the stochastic contract with the wage structure

$w_1 < w_2 = w_3 = w_4$, the principal's cost is given by

$$C_r = c + \frac{(\lambda-1)[q_H + p_1(1-q_H)][1-q_H]c}{(q_H-q_L)[1-(\lambda-1)(1-q_H-q_L-p_1(2-q_H-q_L))]}$$

Differentiation of C_r with respect to p_1 yields

$$\frac{\partial C_r}{\partial p_1} = \frac{c(\lambda-1)(1-q_H)}{q_H-q_L} \cdot \frac{(1-q_H)[1-(\lambda-1)(1-q_H-q_L-p_1(2-q_H-q_L))] - [q_H+p_1(1-q_H)](\lambda-1)(2-q_H-q_L)}{[1-(\lambda-1)(1-q_H-q_L-p_1(2-q_H-q_L))]^2}$$
$$= \frac{c(\lambda-1)(1-q_H)[2-q_H-q_L-\lambda(1-q_L)]}{(q_H-q_L)[1-(\lambda-1)(1-q_H-q_L-p_1(2-q_H-q_L))]^2}$$

Obviously, $\partial C_r/\partial p_1 < 0$ for all p_1 as $\lambda > \frac{2-q_H-q_L}{1-q_L}$. The principal can always achieve a lower cost by increasing p_1 close to 1, i.e., the probability of bonus is almost 1. However, p_1 can not reach 1 due to the incentive constraint. Hence, the second-best optimal stochastic contract does not exist.

PROOF OF PROPOSITION 3:

Suppose (A1) holds, $u''(\cdot) = 0$, and $w \geq 0$. Then, the second-best optimal stochastic contract exists. The optimal stochastic contract pays $b^*(p_1^*)$ with probability one when the good signal is realized and with probability p_1^* when the bad signal is realized. The optimal $p_1^* \in (0,1)$ is given by

$$p_1^* = \max\{p', p''\}$$

where $p' = \frac{1}{1-q_H}\left(\sqrt{1 - \frac{\lambda}{\lambda-1} \cdot \frac{1-q_H}{2-q_H-q_L}} - q_H\right)$ *and* $p'' = 1 - \frac{1}{(\lambda-1)(1-q_L)}$

Consider a stochastic contract of the form

$$w_i = \begin{cases} \underline{w} + b & \text{for } i > 1 \\ \underline{w} & \text{for } i = 1 \end{cases}$$

where $b > 0$. Let f_1^H and f_1^L be the probability of getting a bonus conditional on the agent's high and low action respectively, i.e., $f_1^H = P[i > 1|a_H] = q_H + p_1(1-q_H)$ and $f_1^L = P[i > 1|a_L] = q_L + p_1(1-q_L)$.

The principal's problem becomes

$$\min_{\underline{w},b} \; \underline{w} + f_1^H b$$

subject to

$$\text{(PC)} \qquad \underline{w} + f_1^H b - (\lambda - 1) b f_1^H (1 - f_1^H) \geq c$$

$$\text{(IC)} \qquad b(f_1^H - f_1^L) - (\lambda - 1) b [f_1^H (1 - f_1^H) - f_1^L (1 - f_1^L)] \geq c$$

$$\text{(LL)} \qquad \underline{w} \geq 0$$

Notice first that the (IC) constraint is binding. Suppose, by contradiction, (IC) is slack. Reducing b and increasing $\underline{w}$ by a small amount such that (PC) remains unchanged, the principal decreases the expected payment without violating (LL) or (IC) constraint.

Notice also that the (LL) constraint is binding. Suppose, by contradiction, $\underline{w} > 0$ is the optimal wage scheme. If (PC) does not bind, by reducing $\underline{w}$ by a small amount ϵ, the principal decreases the expected payment without changing (IC) or violating (LL) constraint. If (PC) binds, we can show by contradiction that $p_1 > 0$ implies $\underline{w} = 0$. Thus, $\underline{w}^* = 0$.

Assume that the optimal deterministic contract exists, then the constraint set for the above stochastic contract is non-empty.[11] Thus, at optimum, the bonus is given by

$$b^* = \frac{c}{(f_1^H - f_1^L)[1 - (\lambda - 1)(1 - f_1^H - f_1^L)]}$$

Under the stochastic contract of this form, the principal's cost, $C_r = \underline{w}^* + f_1^H b^* = f_1^H b^*$, is given by

$$C_r = \frac{(q_H + p_1(1 - q_H))c}{(q_H - q_L)(1 - p_1)[1 - (\lambda - 1)(1 - q_H - q_L - p_1(2 - q_H - q_L))]}$$

The principal's cost under the optimal deterministic contract with limited liability

[11] If the deterministic contract has no solution, the dominance of the stochastic contract is trivial. The reason is that the principal can always implement a_H under the stochastic contract by setting $p_1 \in [1/2, 1)$ (Lemma 1). On the other hand, if the optimal deterministic contract exists, i.e., $(\lambda - 1)(1 - q_H - q_L) < 1$, it follows that $(\lambda - 1)(1 - q_H - q_L - p_1(2 - q_H - q_L)) < 1$. Thus, the constraint set under the stochastic contract is non-empty for all p_1.

is given by

$$C_d = \begin{cases} \frac{q_H c}{(q_H - q_L)[1-(\lambda-1)(1-q_H-q_L)]} & \text{if } \frac{1}{\lambda-1} > 1 - q_L \\ c + \frac{(\lambda-1)q_H(1-q_H)c}{(q_H-q_L)[1-(\lambda-1)(1-q_H-q_L)]} & \text{if } \frac{1}{\lambda-1} \leq 1 - q_L \end{cases}$$

Case 1 (LL binds, PC slack): $\frac{1}{\lambda-1} > 1 - q_L$

Note that if $p_1 = 0$, then $C_r = C_d$

Differentiating C_r with respect to p_1 yields

$$\frac{\partial C_r}{\partial p_1} = \frac{c}{q_H-q_L} \frac{1-(\lambda-1)(1-q_H-q_L+q_H(2-q_H-q_L))+p_1 2q_H(\lambda-1)(2-q_H-q_L)+p_1^2(1-q_H)(\lambda-1)(2-q_H-q_L)}{(1-p_1)^2[1-(\lambda-1)(1-q_H-q_L-p_1(2-q_H-q_L))]^2}$$

The stochastic contract reduces the principal's cost, i.e., $C_d > C_r$ if

$$\text{(A1)} \qquad \left.\frac{\partial C_r}{\partial p_1}\right|_{p_1=0} < 0 \Leftrightarrow \frac{1}{\lambda-1} < (2-q_H-q_L)(1+q_H) - 1$$

Provided that (A1) holds, there exists a stochastic contract of the wage structure $w_1 < w_2 = w_3 = w_4$ that strictly dominates the optimal deterministic contract.[12] Solving for the first order condition, I obtain the optimal p_1^*

$$p_1^* = \frac{1}{1-q_H}\left(\sqrt{1 - \frac{\lambda}{\lambda-1}\cdot\frac{1-q_H}{2-q_H-q_L}} - q_H\right)$$

Given (A1) holds, $p_1^* \in (0,1)$. The second-best optimal stochastic contract is characterized by $\underline{w}^* = 0$, $b^*(p_1^*)$, and p_1^*.

Case 2 (LL slack, PC binds): $\frac{1}{\lambda-1} \leq 1 - q_L$

Consider the case when PC binds at the optimum for the stochastic contract.[13]

[12] Analogous to the proof of Lemma 2, it can be shown that under limited liability, adding noise to the good outcome is weakly dominated by the optimal deterministic contract. The cost of a stochastic contract with the wage structure $w_1 = w_2 = w_3 < w_4$ under limited liability is given by $C = \frac{q_H c}{(q_H-q_L)(1-(\lambda-1)(1-p_2 q_H - p_2 q_L))}$, which is weakly larger than C_d – the cost under the optimal deterministic contract – for all $p_2 \in [0,1]$. Thus, the second-best optimal stochastic contract has the wage structure of $w_1 < w_2 = w_3 = w_4$.

[13] If PC does not bind at the optimum. Let p'' be the probability that PC binds and p_1^* be the optimal proability. It follows that $C_r(p'') > C_r(p_1^*)$. Following the similar argument in Case 2, we can show that $C_d^* > C_r(p'')$. Thus, $C_d^* > C_r^*$.

From the binding PC, the principal's cost can be written as

$$C_r = c + \frac{(\lambda-1)[q_H + p_1(1-q_H)](1-q_H)(1-p_1)c}{(q_H-q_L)(1-p_1)[1-(\lambda-1)(1-q_H-q_L-p_1(2-q_H-q_L))]}$$

From Proposition 1, we know that $C_r < C_d$ if and only if $\lambda - 1 > \frac{1-q_H}{1-q_L}$, which always holds given that $\frac{1}{\lambda-1} \leq 1 - q_L$. This completes the proof.

PROOF OF PROPOSITION 4:

Consider finite signals $s \in \{1, \ldots, N\}$. Suppose $u''(\cdot) = 0$ and $\lambda - 1 > \frac{1-q_H^*}{1-q_L^*}$, where q_H^* and q_L^* are the probability of getting the high wage conditional on a_H and a_L under the optimal deterministic contract. Then, there exists a stochastic contract that strictly dominates the optimal deterministic contract.

Suppose that the optimal deterministic contract exists. Because $u''(\cdot) = 0$, the optimal deterministic contract is a binary wage payment. Let $s \in S^- = \{1, \ldots, k\}$ be the set of bad signals that pays out a low wage and $s \in S^+ = \{k+1, \ldots, N\}$ be the set of good signals that pays out a high wage under the optimal deterministic contract. Denote q_H^* and q_L^* the probability of getting the high wage conditional on a_H and a_L under the optimal deterministic contract, i.e., $q_H^* = \sum_{s=k+1}^N q_s^H$ and $q_L^* = \sum_{s=k+1}^N q_s^L$.

Consider a stochastic contract that pays a high wage $w_H = \underline{w} + b$, where $b > 0$ for all $s \in S^+ = \{k+1, \ldots, N\}$ and adds a lottery $(p_1, 1-p_1)$ that pays the same high wage $w_H = \underline{w} + b$ with probability p_1 or a low wage $w_L = \underline{w}$ with probability $1 - p_1$ if any signal $s \in S^- = \{1, \ldots, k\}$ is realized.

Under this stochastic contract, let f_H and f_L be the probability of getting a bonus b conditional a_H and a_L respectively, i.e., $f_H = q_H^* + p_1(1-q_H^*)$ and $f_L = q_L^* + p_1(1-q_L^*)$. Considering $p_1 \in [1/2, 1)$, the non-emptiness of the constraint set follows analogously from Lemma 1 and the strict dominance of the stochastic contract follows analogously from Proposition 1.

PROOF OF PROPOSITION 5:

Suppose $u''(\cdot) < 0$ and $\lambda > 1$. For any degree of relative risk aversion π, there exists a sufficiently large λ such that a stochastic contract with the wage structure $w_1 < w_2 = w_3 = w_4$ strictly dominates the optimal deterministic contract.

Consider a risk-averse agent whose consumption utility is given by $u(w) = \frac{w^{1-\pi}-1}{1-\pi}$, with $\pi < 1$. More precisely, the parameter π measures the agent's degree of relative risk aversion.

As the cost of implementing a stochastic contract increases with the degree of risk aversion, it is sufficient to check if the proposition holds for a very large degree of risk aversion, i.e. $\pi \to 1$. In the limit, as $\pi \to 1$, the consumption utility is approximated by $u(w) = \ln(w)$. Put differently, the wage that the principal offers the agent to obtain utility u is given by $h(u) = e^u$.

I will now consider the minimum cost achieved by a stochastic contract with the wage structure $w_1 < w_2 = w_3 = w_4$ and argue that for a sufficiently large λ, the cost under the stochastic contract is strictly lower than that under the optimal deterministic contract.

Consider a stochastic contract of the form

$$u_i = \begin{cases} \underline{u} + b & \text{for } i > 1 \\ \underline{u} & \text{for } i = 1 \end{cases}$$

where $b > 0$. Analogous to the analysis in Proposition 1, the optimal stochastic contract is characterized by

$$b = \frac{c}{(f_1^H - f_1^L)[1 - (\lambda - 1)(1 - f_1^H - f_1^L)]}$$
$$\text{and } \underline{u} = \frac{-cf_1^L[1 - (\lambda - 1)(1 - f_1^L)]}{(f_1^H - f_1^L)[1 - (\lambda - 1)(1 - f_1^H - f_1^L)]}$$

The principal's cost is given by $C_r = f_1^H e^{\underline{u}+b} + (1 - f_1^H)e^{\underline{u}}$. The minimum cost that the principal can achieve under the stochastic contract is given by

$$\lim_{p_1 \to 1} C_r = 1 \cdot e^{\frac{c(1-q_L)}{q_H - q_L}} + 0 \cdot e^{-\infty} = e^{\frac{c(1-q_L)}{q_H - q_L}}$$

Suppose that the optimal deterministic contract exists. The principal's cost under the optimal deterministic contract is given by $C_d = q_H e^{u_H} + (1 - q_H)e^{u_L}$, where

$$u_L = \frac{-cq_L[1-(\lambda-1)(1-q_L)]}{(q_H-q_L)[1-(\lambda-1)(1-q_H-q_L)]}$$
$$\text{and } u_H = \frac{c(1-q_L)[1+(\lambda-1)q_L]}{(q_H-q_L)[1-(\lambda-1)(1-q_H-q_L)]}$$

As λ gets very large, $\lim_{\lambda\to\infty} C_d = e^{\frac{cq_L(1-q_L)}{(q_H-q_L)(q_H+q_L-1)}}$, which is strictly larger than the minimum cost under the stochastic contract.

PROOF OF PROPOSITION 6:

Suppose the agent exhibits disappointment aversion according to Bell (1985), $u''(\cdot)=0$, and $\lambda-1>\frac{1-q_H}{1-q_L}$. Consider two actions and two signals. Then, there exists a stochastic contract with the wage structure $w_1<w_2=w_3=w_4$ that strictly dominates the optimal deterministic contract.

The proof of Proposition 6 closely follows the proof of Proposition 1. I first show that the principal's problem remains the same regardless of whether the agent exhibits disappointment aversion (Bell, 1985) or loss aversion (Kőszegi and Rabin, 2006, 2007).

Consider a stochastic contract of the form

$$w_i = \begin{cases} \underline{w}+b & \text{for } i>1 \\ \underline{w} & \text{for } i=1 \end{cases}$$

where $b>0$. Let f_1^H and f_1^L be the probability of getting a bonus conditional on the agent's high and low action respectively, i.e., $f_1^H = P[i>1|a_H] = q_H + p_1(1-q_H)$ and $f_1^L = P[i>1|a_L] = q_L + p_1(1-q_L)$.

Under the disappointment aversion, the agent compares a realized outcome to the certainty equivalence of the prospect, which is given by $\text{CE}_r(a_H)=\underline{w}+f_1^H b$. With probability f_1^H a bonus is realized, the agent feels elated by receiving $(1-f_1^H)b$ more than the certainty equivalence. With probability $(1-f_1^H)$ a bonus is not realized, the agent feels disappointed by receiving $f_1^H b$ less than the certainty equivalence. The agent's utility from choosing a_H is given by

$$\underline{w}+f_1^H b+f_1^H(1-f_1^H)b-\lambda(1-f_1^H)f_1^H b = \underline{w}+f_1^H b-(\lambda-1)f_1^H(1-f_1^H)b$$

The (IC) constraint is given by

$$b(f_1^H - f_1^L) - (\lambda - 1)b[f_1^H(1 - f_1^H) - f_1^L(1 - f_1^L)] = c$$

Notice that the above (PC) and (IC) constraints coincide with the principal's constraints under CPE loss aversion.

Assume w.l.o.g. $1 > p_1 \geq 1/2$, the non-emptiness of the constraint set follows from Lemma 1, and the dominance of the stochastic contract analogously follows from Proposition 1.

PROOF OF PROPOSITION 7:

Suppose the agent exhibits the PPE loss aversion, $u''(\cdot) = 0$, $q_H + 2q_L \leq 2$ *and* $\lambda - 1 > \frac{1-q_H}{1-q_L}$. *Consider two actions and two signals. Then, there exists a stochastic contract with the wage structure* $w_1 < w_2 = w_3 = w_4$ *that strictly dominates the optimal deterministic contract.*

Consider a stochastic contract of the form

$$w_i = \begin{cases} \underline{w} + b & \text{for } i > 1 \\ \underline{w} & \text{for } i = 1 \end{cases}$$

where $b > 0$. Let f_1^H and f_1^L be the probability of getting a bonus conditional on the agent's high and low action respectively, i.e., $f_1^H = P[i > 1|a_H] = q_H + p_1(1 - q_H)$ and $f_1^L = P[i > 1|a_L] = q_L + p_1(1 - q_L)$.

Under PPE loss aversion, the agent identifies (i) the set of personal equilibrium (PE) that includes all actions the agent can follow through, and (ii) the preferred action among the set of personal equilibrium (PPE).

$$a \in \text{PE} \Leftrightarrow EU(a|a) \geq EU(a'|a)\ \forall a' \neq a$$
$$a \in \text{PPE} \Leftrightarrow EU(a|a) \geq EU(a'|a')\ \forall a' \in \text{PE}$$

For $a_H \in$ PE, $EU(a_H|a_H) \geq EU(a_L|a_H)$, the latter refers to the expected utility when the agent expects to choose a_H but actually chooses a_L, is given by

$$\underline{w} + f_1^H b - (\lambda - 1)f_1^H(1 - f_1^H)b - c \geq \underline{w} + f_1^L b + f_1^L(1 - f_1^H)b - \lambda(1 - f_1^L)f_1^H b + c$$

This is equivalent to

$$(a_H\text{-PE}) \qquad b \geq \frac{2c}{(f_1^H - f_1^L)[2 + f_1^H(\lambda - 1)]} := \underline{b}$$

Analogously, for $a_L \in$ PE

$$(a_L\text{-PE}) \qquad b \leq \frac{(\lambda + 1)c}{(f_1^H - f_1^L)[2 + f_1^L(\lambda - 1)]} := \bar{b}$$

Note that $\bar{b} > \underline{b}$ for all $\lambda \geq 1$.

The principal's problem becomes

$$\min_{\underline{w},b} \ \underline{w} + f_1^H b$$

subject to

$$(\text{PC}) \qquad \underline{w} + f_1^H b - (\lambda - 1) b f_1^H (1 - f_1^H) = c$$

$$(a_H\text{-PPE}) \qquad b \geq \frac{c}{(f_1^H - f_1^L)[1 - (\lambda - 1)(1 - f_1^H - f_1^L)]} := \tilde{b}$$

$$(a_H\text{-PE}) \qquad b \geq \underline{b}$$

Assume that the optimal deterministic contract exists, it follows that the principal's constraint set for the stochastic contract is non-empty. There exists $p_1 \in [0, 1)$ such that $\tilde{b} \geq \underline{b}$. Consider a relaxed problem without (a_H-PE) constraint. The relaxed problem coincides with the principal's problem of CPE loss aversion and, from Proposition 1, the cost is given by

$$C_r = c + \frac{(\lambda - 1)[q_H + p_1(1 - q_H)][1 - q_H]c}{(q_H - q_L)[1 - (\lambda - 1)(1 - q_H - q_L - p_1(2 - q_H - q_L))]}$$

Following the above analysis analogously, if $q_H + 2q_L \leq 2$, then the principal's cost under the optimal deterministic contract is given by

$$C_d = c + \frac{(\lambda - 1)q_H(1 - q_H)c}{(q_H - q_L)[1 - (\lambda - 1)(1 - q_H - q_L)]}$$

Because $\lambda - 1 > \frac{1 - q_H}{1 - q_L}$, $C_r < C_d$. This completes the proof.